Firm Entry, Excess Capacity and Endogenous Productivity
Supplementary Appendix

Anthony Savagar   Huw Dixon

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This supplementary appendix presents a stripped-down version of the model; presents the households Hamiltonian; elaborates on the role of the labor market with firm entry and increasing marginal costs; and also outlines the solution to the 4d dynamical system. This provides deeper understanding of the short-run and long-run behaviour of primitive variables (C, e, K, n). Additionally the appendix provides detailed derivations of Jacobian minors in general, derives steady state for the parameterized case, and verifies general results for the parameterized case. The parameterized results help intuition.

A Simple Model with Constant Marginal Cost

This section introduces the main idea of the paper in a partial equilibrium model with constant marginal costs and no capital. A firm produces with the following production function

$$y = A\frac{L}{n} - \phi \tag{1}$$

It pays an overhead cost ϕ and employs labor L which is divided equally among all n firms. We impose symmetry which implies aggregate variables are n multiplied by firm level variables. Assuming a constant returns to scale aggregator, aggregate output is the number of producers multiplied by their production $Y = AL - n\phi$, and defining productivity as output per unit of labor gives

$$\mathcal{P} = \frac{Ay}{y + \phi} \tag{2}$$

which implies productivity is increasing in output due to the overhead cost $\mathcal{P}_y = \frac{A\phi}{(y(t)+\phi)^2} > 0$.[1] The effect of an additional firm on aggregate output is $Y_n = y + ny_n$. The first term is the entrants own output contribution (1) (the extensive margin contribution), and the second term $ny_n = -A\frac{L}{n}$ represents aggregate business stealing (the intensive margin contribution). Therefore the overall effect of an entrant on aggregate output is $Y_n = -\phi$ which is negative and equal to the fixed cost an entrant incurs.

Per firm profits are revenue less costs, where output price is the numeraire, $\pi = y - w\frac{L}{n}$. Under imperfect competition wages are a fraction of marginal products $w = (1-\zeta)A$ where $\zeta \in (0, 1)$ is the Lerner Index of market power, and A is the marginal product of labor.[2] At this imperfectly competitive wage, profits are increasing in market power as costs diminish:

$$\pi = y - (1 - \zeta)(y + \phi). \tag{3}$$

The free-entry condition on firm dynamics implies that in the long-run firms enter the market to arbitrage profits to zero which gives steady state output and labor per firm

$$y^* = \frac{1-\zeta}{\zeta}\phi, \qquad \frac{L^*}{n^*} = \frac{\phi}{A\zeta}, \qquad \zeta \in (0, 1),\ A > 0 \tag{4}$$

so steady-state productivity is $\mathcal{P}^* = (1-\zeta)A$. Firms' output is decreasing in market power because with an increasing markup between price and marginal cost firms can produce less and still cover their fixed cost of production in zero profit equilibrium. Per firm production (firm size) is independent of

[1] Subscripts denote total derivatives e.g. $\mathcal{P}_y = \frac{d\mathcal{P}}{dy}$.

[2] The Lerner index measures the difference between price and marginal cost as a proportion of price $\zeta = \frac{p-mc}{p} \in (0, 1)$. In related literature, macroeconomists have often preferred the markup notation $\mu = \frac{p}{mc} \in (1, \infty)$, implying $w = \frac{1}{\mu}A$.

technology A in the long run. An improvement in technology raises profits and therefore entry until all firms return to producing the same output. The intensive margin y^* is fixed, but the extensive margin $Y^* = n^* y^*$ adjusts by the change in number of firms. From (2) the effect of a technology shock on productivity is

$$\mathcal{P}_A(t) = \frac{\mathcal{P}(t)}{A} + \mathcal{P}_y(t) y_A(t) \tag{5}$$

so it depends on the direct effect of a shift in the level of technology $\frac{\mathcal{P}(t)}{A} = \frac{y(t)}{y(t)+\phi}$ and it depends on the indirect response of output per firm y_A interacted with returns to scale $\mathcal{P}_y$. Therefore if the economy begins in steady state $y(0) = y^*$, before a new technological advancement, the short-run effect of the shock evaluated at steady state is

$$\mathcal{P}_A(0)|^* = \frac{y^*}{y^* + \phi} + \frac{A\phi}{(y^* + \phi)^2} y_A(0)|^* \tag{6}$$

And since it will return to its initial level in the long run $y(t \to \infty) = y^*$ and $y_A(\infty) = y_A^* = 0$ and $\mathcal{P}_A(\infty) = \mathcal{P}^*$, the effect of the shock in the long run is

$$\mathcal{P}_A^* = \frac{y^*}{y^* + \phi} \tag{7}$$

Therefore comparing (6) and (7), shows there is a short-run endogenous productivity effect.

$$\mathcal{P}_A(0)|^* - \mathcal{P}_A^* = \frac{A\zeta^2}{\phi} y_A(0)|^*, \qquad \phi > 0 \tag{8}$$

Equation (8) is analogous to the main result of the paper. It states that, given increasing returns ($\phi > 0$), imperfect competition ($\zeta \in (0,1)$) combined with short-run output variation of incumbents ($y_A(0)|^* \neq 0$) causes endogenous productivity effects in response to a technology shock. The sign of the effect (overshooting or undershooting) depends on output per firm's short-run response. Therefore imperfect competition and short-run output variation are necessary and jointly sufficient conditions. We argue that the short-run intensive margin response ($y_A(0)|^* \neq 0$) arises because of slow firm entry.[3] That is, incumbents' output varies in the short-run absence of entry. But in the long-run, entry will adjust, which causes business stealing, until profits are zero so output per firm returns to its original level.

A.1 Output per Firm Variation

To understand the intertemporal variation in firms' intensive margin which creates endogenous productivity fluctuations, recall that the production function of a firm has one input (labor per firm $\frac{L}{n}$), and thus varies through aggregate labor and total number of firms. For now, assume that labor is determined endogenously $L(A; C(A))$. It depends on technology directly, and indirectly through consumption $C(A)$. Therefore the effect of a technology shock on output per firm comes through three channels:

$$y_A(t) = \frac{L(t)}{n(t)} \left[1 + \varepsilon_{LA}(t) - \varepsilon_{nA}(t)\right] \tag{9}$$

where notation $\varepsilon_{XY} = X_Y \frac{Y}{X}$ is the elasticity of X with respect to Y.[4] In the short run, firms are slow to respond (quasi-fixed) so only labor adjusts

$$y_A(0)|^* = \frac{\phi}{\zeta A} \left[1 + \varepsilon(0)_{LA}|^*\right] \tag{10}$$

therefore the negative business stealing effect of entrants on incumbents' output is not present.[5] The short-run response of output per firm only depends on the short-run elasticity of labor. This will be positive with a dominant substitution effect, or negative with a dominant income effect (assuming leisure is a normal good).[6] Hence from substituting into (8) the size and sign of the endogenous productivity effect depend on market power, and the initial

[3] In general equilibrium we endogenously generate this through an endogenous sunk cost.

[4] The result follows from taking the derivative of $y = A\frac{L}{n} - \phi$ with respect to A: $y_A = \frac{L}{n} + \frac{A}{n^2}[nL_A - Ln_A]$.

[5] Assuming technology creates entry $n_A > 0$, not exit $n_A < 0$.

[6] A sufficiently strong income effect $\varepsilon(0)_{LA}|^* < -1$ reduces output per firm and causes productivity undershooting.

elasticity of labor to technology.

$$\mathcal{P}_A(0)|^* - \mathcal{P}_A^* = \zeta(1 + \varepsilon(0)_{LA}|^*) \tag{11}$$

With instantaneous entry this short-run effect is not present because number of firms respond instantaneously to ensure the change in labor is offset by a change in number of firms such that labor per firm is instantaneously at the scale that causes zero profits in (3).

Our paper formalizes this insight in a more realistic environment, with capital, increasing marginal costs, and microfoundations for slow firm entry and imperfect competition. Unlike the constant marginal costs of this simple example, increasing marginal costs teamed with an overhead cost create an efficient firm scale (full capacity benchmark). Thus we shall view short-run variations in incumbents' intensive margin as *excess capacity utilization.* This is a measurable concept which ties our work to a long-tradition in econometrics and microproduction theory Berndt and Morrison 1981.

B Household Problem

Use the Maximum Principle to obtain the necessary conditions for a solution to the household's utility maximisation problem. The current value Hamiltonian is

$$\begin{aligned} \hat{\mathcal{H}}(t) = u(C(t), L(t)) + \\ \lambda(t)(w(t)L(t) + r(t)K(t) + \Pi(t) - C(t)) \end{aligned} \tag{12}$$

The Pontryagin necessary conditions are

$$\hat{\mathcal{H}}_C(K, L, C, \lambda) = 0 \implies u_C - \lambda = 0 \tag{13}$$

$$\hat{\mathcal{H}}_L(K, L, C, \lambda) = 0 \implies u_L + \lambda w = 0 \tag{14}$$

$$\hat{\mathcal{H}}_K(K, L, C, \lambda) = \rho\lambda - \dot{\lambda} \implies \lambda r = \rho\lambda - \dot{\lambda}$$

$$\implies \frac{\dot{\lambda}}{\lambda} = -(r - \rho) \tag{15}$$

$$\hat{\mathcal{H}}_\lambda := \dot{K} \implies \dot{K} = rK + wL + \Pi - C \tag{16}$$

The costate variable $\lambda(t) = u_C$ is the shadow price of wealth in utility units. Equations (13)-(15) reduce to two equations, the consumption Euler equation (intertemporal condition) and a static relationship between labor and consumption (intratemporal condition) as presented in the paper.

C Labor Market

We model labor in a traditional way as in Bilbiie et al. 2007; Bilbiie, Ghironi, and Melitz 2012; Jaimovich and Floetotto 2008, but the combination of entry with increasing marginal costs $\nu \in (0, 1)$ means that both wage and labor supply are increasing in entry. Conversely, with constant MC (as in above papers) they are unaffected, as number of firms does not affect the efficiency at which labor is employed.[7] We first present general results, and then specify CES preferences as in above papers. This also helps to discern the role of indivisible labor which is popularly assumed.

Turnovsky 2000, ch. 8, p.232 provides a useful discussion of the intratemporal condition without entry or imperfect competition (centralized problem). Whereas here we have $w(C, K, n)$ specified by factor market equilibrium

$$u_L(L) + u_C(C)w(C, K, n) = 0 \tag{17}$$

[7]This appendix discussion is motivated by a comment (Rotemberg 2008) on the labor market in the discussion section of Bilbiie et al. 2007.

where $u_{CC}, u_{LL}, u_L < 0$, $u_C > 0$ and $u_{CL} = u_{LC} = 0$. Take the derivative with respect to C, K, n

$$u_{LL}L_C + u_{CC}w + u_C w_C = 0 \tag{18}$$

$$u_{LL}L_K + u_C w_K = 0 \tag{19}$$

$$u_{LL}L_n + u_C w_n = 0 \tag{20}$$

We need to substitute out the total derivatives of wage, and then collect together the labor responses. First we need the simple labor effect, which the others depend on. Then for capital and firms w_K, w_n there is a direct ceteris paribus effect (partial derivative that holds labor constant), plus an indirect labor supply effect, whereas consumption only affects wage through labor.

$$w(C, K, n) = (1-\zeta)An^{1-\nu}F_L(K, L) \tag{21}$$

$$w_L = (1-\zeta)An^{1-\nu}F_{LL}(K, L) < 0 \tag{22}$$

$$w_C = w_L L_C \tag{23}$$

$$w_K = \frac{\partial w}{\partial K} + w_L L_K \tag{24}$$

$$w_n = \frac{\partial w}{\partial n} + w_L L_n \tag{25}$$

Substitute (23)–(25) into (18)–(20), and collect labor supply responses on the left-hand side. Then use $\frac{\partial w}{\partial K} = w\frac{F_{LK}}{F_L}$, $\frac{\partial w}{\partial n} = (1-\nu)\frac{w}{n}$ and $u_L = -u_C w$.

$$L_C = \frac{-u_{CC}w}{u_{LL} + u_C w_L} < 0 \tag{26}$$
$$= \frac{u_L}{u_{LL} + u_C w_L}\frac{u_{CC}}{u_C} < 0 \tag{27}$$
$$L_K = \frac{-u_C}{u_{LL} + u_C w_L}\frac{\partial w}{\partial K} \tag{28}$$
$$= \frac{u_L}{u_{LL} + u_C w_L}\frac{F_{LK}}{F_L} > 0 \tag{29}$$
$$L_n = \frac{-u_C}{u_{LL} + u_C w_L}\frac{\partial w}{\partial n} \tag{30}$$
$$= \frac{u_L}{u_{LL} + u_C w_L}\frac{1-\nu}{n} > 0, \quad \nu \in (0,1) \tag{31}$$

Therefore the labor supply responses create an opposing effect in the wage responses $w_K,\ w_n$. That is, on the one hand the direct effects ($\partial w/\partial K$ and $\partial w/\partial K$) imply increased capital or firms raises wage because capital complements MPL and firms divide labor more so also enhances MPL since $\nu \in (0,1)$. But on the other hand the stimulus in labor supply reduces wage. However, as shown below the direct effect always dominates, and leads to the conclusion that wage is strictly increasing in the models' primitive variables $w_C,\ w_K,\ w_n > 0$.

$$w_C = \frac{u_L}{u_{LL} + u_C w_L}\frac{u_{CC}}{u_C}w_L > 0 \tag{32}$$
$$w_K = \frac{u_{LL}}{u_{LL} + u_C w_L}\frac{\partial w}{\partial K} \tag{33}$$
$$= \frac{u_{LL}}{u_{LL} + u_C w_L}\frac{wF_{LK}}{F_L} > 0 \tag{34}$$
$$w_n = \frac{u_{LL}}{u_{LL} + u_C w_L}\frac{\partial w}{\partial n} \tag{35}$$
$$= \frac{u_{LL}}{u_{LL} + u_C w_L}(1-\nu)\frac{w}{n} > 0, \quad \nu \in (0,1) \tag{36}$$

C.1 Technology and the Labor Market

As before to get the labor effect, take the derivative of the intratemporal condition with respect to A, substitute in the wage total derivative w_A, and collect L_A on the left-hand side. Then substitute L_A back into the wage total derivative to get the wage effect.

$$u_{LL}L_A + u_{CC}C_A w + u_C w_A = 0 \tag{37}$$

Substitute in

$$w = (1-\zeta)An^{1-\nu}F_L(K,L) \tag{38}$$
$$w_A = \frac{\partial w}{\partial A} + \frac{\partial w}{\partial K}K_A + \frac{\partial w}{\partial n}n_A + w_L L_A \tag{39}$$

Notice that the partial derivative direct effect $\frac{\partial w}{\partial A} = \frac{w}{A}$, excludes the partial direct effect of labor changing $w_L \frac{\partial L}{\partial A}$, even though this is also a direct effect in the sense that it is not felt through endogenous variables C, K, n (this is clear in (47)). Collect terms in L_A:

$$u_{LL}L_A + u_{CC}C_A w + u_C \left[\frac{\partial w}{\partial A} + \frac{\partial w}{\partial K}K_A + \frac{\partial w}{\partial n}n_A + w_L L_A\right] = 0 \tag{40}$$
$$L_A = \frac{-1}{u_{LL} + u_C w_L}\left[u_C \frac{\partial w}{\partial A} + u_{CC} w C_A + u_C \frac{\partial w}{\partial K}K_A + u_C \frac{\partial w}{\partial n}n_A\right] \tag{41}$$

Substitute in the partial derivatives $\frac{\partial w}{\partial A} = \frac{w}{A}$, $\frac{\partial w}{\partial K} = w\frac{F_{LK}}{F_L}$, $\frac{\partial w}{\partial n} = (1-\nu)\frac{w}{n}$ and use $u_L = -u_C w$ gives

$$L_A = \frac{u_L}{u_{LL} + u_C w_L}\left[\frac{1}{A} + \frac{u_{CC}}{u_C}C_A + \frac{F_{LK}}{F_L}K_A + \frac{(1-\nu)}{n}n_A\right] \tag{42}$$

which using our labor total derivatives derived above, and defining the direct effect $\frac{\partial L}{\partial A} \equiv \frac{u_L}{u_{LL}+u_C w_L}\frac{1}{A} = \frac{-u_C}{u_{LL}+u_C w_L}\frac{\partial w}{\partial A}$, can be compactly written

$$L_A = \frac{\partial L}{\partial A} + L_n n_A + L_K K_A + L_C C_A \tag{43}$$

or in terms of elasticities, where $x_y \frac{y}{x} = \varepsilon_{xy}$ is the elasticity of x with respect to y, and $-\frac{Cu_{CC}}{u_C} = \sigma(C)$ is the coefficient of relative risk aversion (CRRA)

$$\varepsilon_{LA} = \frac{1}{L} \frac{u_L}{u_{LL} + u_C w_L} \left[1 - \sigma(C)\varepsilon_{CA} + \frac{KF_{LK}}{F_L}\varepsilon_{KA} + (1-\nu)\varepsilon_{nA} \right] \tag{44}$$

As before, the total labor derivative can be substituted back into the total derivative for wage

$$w_A = \frac{\partial w}{\partial A} + \frac{\partial w}{\partial K} K_A + \frac{\partial w}{\partial n} n_A + w_L \left(\frac{\partial L}{\partial A} + L_n n_A + L_K K_A + L_C C_A \right) \tag{45}$$

$$= \frac{\partial w}{\partial A} + w_L \frac{\partial L}{\partial A} + \left(\frac{\partial w}{\partial K} + w_L L_K \right) K_A + \left(\frac{\partial w}{\partial n} + w_L L_n \right) n_A + w_L L_C C_A \tag{46}$$

$$= \frac{\partial w}{\partial A} + w_L \frac{\partial L}{\partial A} + w_K K_A + w_n n_A + w_C C_A \tag{47}$$

Then substituting back in the total wage derivative coefficients

$$= \frac{\partial w}{\partial A} \left(1 - \frac{w_L u_C}{u_{LL} + u_C w_L} \right) + \frac{1}{u_{LL} + u_C w_L} \left[u_{LL} \frac{w F_{LK}}{F_L} K_A + u_{LL}(1-\nu)\frac{w}{n} n_A + w_L u_L \frac{u_{CC}}{u_C} C_A \right] \tag{48}$$

$$= \frac{1}{u_{LL} + u_C w_L} \left[u_{LL} \left(\frac{\partial w}{\partial A} + \frac{w F_{LK}}{F_L} K_A + (1-\nu)\frac{w}{n} n_A \right) + w_L u_L \frac{u_{CC}}{u_C} C_A \right] \tag{49}$$

$$= \frac{w}{A} \frac{1}{u_{LL} + u_C w_L} \left[u_{LL} \left(1 + \frac{F_{LK}}{F_L} A K_A + (1-\nu)\frac{A}{n} n_A \right) + u_L A \frac{F_{LL}}{F_L} \frac{u_{CC}}{u_C} C_A \right] \tag{50}$$

In terms of elasticities and CRRA:

$$\varepsilon_{wA} = \frac{1}{u_{LL} + u_C w_L} \left[u_{LL} \left(1 + \frac{F_{LK}}{F_L} K \varepsilon_{KA} + (1-\nu)\varepsilon_{nA} \right) - u_L \frac{F_{LL}}{F_L} \sigma(C) \varepsilon_{CA} \right] \tag{51}$$

C.2 Parameterized Labor Market

The following are useful in the total derivatives for labor L_x and wage w_x (see Section E for the full functional forms)

$$\frac{u_L}{u_{LL} + u_C w_L} = \frac{1}{1+\eta-\beta} L \tag{52}$$
$$\frac{u_{LL}}{u_{LL} + u_C w_L} = \frac{\eta}{1+\eta-\beta} \tag{53}$$

Since $w = (1-\zeta)AF_l = (1-\zeta)A\beta k^\alpha l^{\beta-1} = (1-\zeta)A\beta K^\alpha L^{\beta-1} n^{1-\nu}$ and by the intratemporal condition $w = -\frac{u_L}{u_C} = \xi L^\eta C^\sigma$, which looks like wage is increasing in labor, unless we note wage implicitly depends on L by the first relationship. Therefore $(1-\zeta)A\beta K^\alpha L^{\beta-1} n^{1-\nu} = \xi L^\eta C^\sigma$ and the intratemporal condition and its derivatives are

$$L(C,K,n) = \left(\frac{(1-\zeta)AK^\alpha \beta n^{1-(\alpha+\beta)}}{\xi C^\sigma}\right)^{\frac{1}{1+\eta-\beta}} \tag{54}$$
$$L_n = \left(\frac{1-\nu}{1+\eta-\beta}\right)\frac{L}{n} > 0, \quad \nu \in (0,1) \tag{55}$$
$$L_K = \left(\frac{\alpha}{1+\eta-\beta}\right)\frac{L}{K} > 0 \tag{56}$$
$$L_C = -\left(\frac{\sigma}{1+\eta-\beta}\right)\frac{L}{C} < 0 \tag{57}$$

Substituting back in for wage gives

$$w(C, L(C,K,n)) = -\frac{u_L}{u_C} = \xi L(C,K,n)^\eta C^\sigma \tag{58}$$
$$w(C,K,n) = \left[(\xi C^\sigma)^{1-\beta}((1-\zeta)AK^\alpha \beta n^{1-\nu})^\eta\right]^{\frac{1}{1+\eta-\beta}} \tag{59}$$

Therefore the wage derivatives are

$$w_C = \frac{(1-\beta)\sigma}{1+\eta-\beta}\frac{w}{C} \tag{60}$$

$$w_K = \left(\frac{\eta}{1+\eta-\beta}\right)\alpha\frac{w}{K} \tag{61}$$

$$w_n = \left(\frac{\eta}{1+\eta-\beta}\right)(1-\nu)\frac{w}{n} \tag{62}$$

We can plug these total derivatives into the general technology response equations L_A (43) and w_A (47) to understand how technology affects the labor market, and this corroborates the taking derivatives of (54) and (59) directly.

$$L_A = \frac{L}{1+\eta-\beta}\left[\frac{1}{A} - \frac{\sigma}{C}C_A + \frac{\alpha}{K}K_A + \frac{(1-\nu)}{n}n_A\right] \tag{63}$$

which in terms of elasticities gives:

$$L_A\frac{A}{L} = \frac{1}{1+\eta-\beta}\left[1 - \sigma\frac{A}{C}C_A + \alpha\frac{A}{K}K_A + (1-\nu)\frac{A}{n}n_A\right] \tag{64}$$

$$\varepsilon_{LA} = \frac{1}{1+\eta-\beta}\left[1 - \sigma\varepsilon_{CA} + \alpha\varepsilon_{KA} + (1-\nu)\varepsilon_{nA}\right] \tag{65}$$

And following the same process for wage, where the direct partial derivative effect of changing A on (59) is $\frac{\eta}{1+\eta-\beta}\frac{w}{A}$, which combines the partial direct wage effect and partial direct labor effect $\frac{\partial w}{\partial A} + w_L\frac{\partial L}{\partial A} = \frac{\eta}{1+\eta-\beta}\frac{w}{A}$.[8]

$$w_A = \frac{\partial w}{\partial A} + w_L\frac{\partial L}{\partial A} + w_K K_A + w_n n_A + w_C C_A \tag{66}$$

$$\begin{aligned} w_A = \frac{w}{A}\Bigg[&\frac{\eta}{1+\eta-\beta} + \frac{\eta}{1+\eta-\beta}\alpha\frac{A}{K}K_A \\ &+\frac{\eta}{1+\eta-\beta}(1-\nu)\frac{A}{n}n_A + \frac{(1-\beta)\sigma}{1+\eta-\beta}\frac{A}{C}C_A\Bigg] \end{aligned} \tag{67}$$

[8]This keeps consistency with the general results notation, even though the partial derivative of (59) is clearly $\frac{\eta}{1+\eta-\beta}\frac{w}{A}$ and we might label it $\frac{\partial w}{\partial A}$. This would be erroneous relative to earlier notation as (59) implicitly includes the direct partial labor effect $w_L\frac{partialL}{\partial A}$ as labor is substituted out.

Or in terms of elasticities

$$\varepsilon_{wA} = \frac{1}{1+\eta-\beta}\left[\eta(1+\alpha\varepsilon_{KA}+(1-\nu)\varepsilon_{nA})+(1-\beta)\sigma\varepsilon_{CA}\right] \tag{68}$$

The results show that firm entry positively affects labor supply and wages in response to technology when there are increasing marginal costs $\nu \in (0,1)$, but is ineffective with constant returns $\nu = 1$. Even with non-constant returns, the effect on wages is lost if there is indivisible labor $\eta = 0$, which is a popular assumption in related literature Bilbiie, Ghironi, and Melitz 2012; Jaimovich and Floetotto 2008. Indivisible labor implies only the response of consumption affects wages.

C.2.1 Technology in Short-run (t=0)

In the short-run $K_A = n_A = 0$ hence

$$\varepsilon_{LA}(0) = \frac{1}{L}\frac{u_L}{u_{LL}+u_C w_L}\left[1-\sigma(C)\varepsilon_{CA}(0)\right] \tag{69}$$

$$\varepsilon_{wA}(0) = \frac{1}{u_{LL}+u_C w_L}\left[u_{LL}-u_L\frac{F_{LL}}{F_L}\sigma(C)\varepsilon_{CA}(0)\right] \tag{70}$$

D Dynamics (Jacobian)

Superficially the Jacobian and related results are analogous to Brito and Dixon 2013 which studies the model's dynamics with perfect competition, $\zeta = 0$. By analagous, we mean the elements of the Jacobian matrix have the same economic interpretation. However, the extension to imperfect competition $\zeta > 0$, which our theory of excess capacity utilization depends on, exceptionally complicates the mathematics, and alters the relative sizes of the Jacobian's elements and its symmetry. Loss of symmetry makes the characteristic polynomial less tractable. With perfect competition, the model is solvable as a centralized planner problem, so the four dynamic equations result from a single optimal control problem with two constraints. Consequently the dynamical system has symmetry properties that are well-studied in the theory of optimal control Feichtinger, Novak, and Wirl 1994; Dockner and Feichtinger 1991.[9] This yields four closed-form eigenvalues of the form $a \pm b \pm c$ (hence two symmetries) and consequently solutions. However, with imperfect competition, the dynamical system arises from a decentralized problem of the household and the firm, therefore it loses the symmetric properties that arise from a single dynamic optimization problem.

The reason for our tractable analysis is to formalize quasi-fixity of capital and number of firms $n_A(0)|^* = K_A(0)|^* = 0$, which forms the main bridge in our proof of Theorem 1. This approach emphasizes the importance of short-run variations in labor supply, driven by the short-run consumption jump $C_A(0)|^*$ that puts the economy on its stable manifold.

The Jacobian is defined as follows with each element evaluated at steady

[9] Analytical studies of 4d systems are uncommon. Some useful references are Bhandari, Haque, and Turnovsky 1990; Heijdra and Ligthart 2010; Chatterjee, Sakoulis, and Turnovsky 2003; Shi and Epstein 1993; Turnovsky 2000.

state.

$$\mathbf{J} = \left. \begin{bmatrix} \dot{C}_C & \dot{C}_e & \dot{C}_K & \dot{C}_n \\ \dot{e}_C & \dot{e}_e & \dot{e}_K & \dot{e}_n \\ \dot{K}_C & \dot{K}_e & \dot{K}_K & \dot{K}_n \\ \dot{n}_C & \dot{n}_e & \dot{n}_K & \dot{n}_n \end{bmatrix} \right|^* = \left. \begin{bmatrix} \frac{C}{\sigma(C)} r_C & 0 & \frac{C}{\sigma(C)} r_K & \frac{C}{\sigma(C)} r_n \\ -\frac{\pi_C}{\gamma} & \rho & -\frac{\pi_K}{\gamma} & -\frac{\pi_n}{\gamma} \\ Y_C - 1 & 0 & Y_K & Y_n \\ 0 & 1 & 0 & 0 \end{bmatrix} \right|^* \tag{71}$$

$$\left. \begin{bmatrix} \rho - (1-\zeta) Y_K & 0 & \frac{C}{\sigma(C)} r_K & \frac{y}{\sigma(C)} (1-\zeta)(1-\nu) Y_K \\ -\frac{Y_C(1-(1-\zeta)\nu)}{n\gamma} & \rho & -\frac{Y_K(1-(1-\zeta)\nu)}{n\gamma} & -\frac{(Y_n - y)(1-(1-\zeta)\nu)}{n\gamma} \\ Y_C - 1 & 0 & Y_K & Y_n \\ 0 & 1 & 0 & 0 \end{bmatrix} \right|^* \tag{72}$$

$$\begin{bmatrix} \frac{y^*}{\sigma}(1-\zeta) A F_{kl} L_C & 0 & \frac{y^*}{\sigma}(1-\zeta) A (F_{kk} + F_{kl} L_K) & \frac{y^*}{\sigma}(1-\zeta) \left[(1-\nu) \frac{\rho}{1-\zeta} + A F_{kl} L_n \right] \\ -\frac{(1-(1-\zeta)\nu)}{\gamma n^*} A F_l L_C & \rho & -\frac{(1-(1-\zeta)\nu)}{\gamma n^*} \left(\frac{\rho}{1-\zeta} + A F_l L_K \right) & \frac{1}{\gamma n^*} (\nu\phi - (1-(1-\zeta)\nu) A F_l L_n) \\ A F_l L_C - 1 & 0 & \frac{\rho}{1-\zeta} + A F_l L_K & \frac{-\zeta\nu\phi}{1-(1-\zeta)\nu} + A F_l L_n \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{73}$$

Entry only affects dynamics (rates of change) through entry $\dot{n}$ and rate of entry $\dot{e}$ (see column 2). Entry $\dot{n}$ is affected by entry only (row 4). Whereas the rate of entry $\dot{e}$ (row 2) is affected by all model variables. Therefore, entry affects little (column 2), but the stock of firms affects $\dot{C}, \dot{e}, \dot{K}$ (column 4).

D.1 Jacobian Elements

D.1.1 Output

$$Y = ny \tag{74}$$

$$Y_C = n y_C < 0 \tag{75}$$

$$Y_K = n y_K > 0 \tag{76}$$

$$Y_n = y + n y_n \lesseqgtr 0 \tag{77}$$

D.1.2 Output Per Firm

The total derivative of K and n includes both the direct partial derivative effect, and the indirect effect through labor.

$$y = An^{-\nu}F(K,L) - \phi \tag{78}$$

$$y_C = An^{-\nu}F_L L_C = An^{-\nu}F_L \frac{u_L}{u_{LL} + u_C w_L}\frac{u_{CC}}{u_C} < 0 \tag{79}$$

$$= -An^{-\nu}F_L \frac{u_{CC}w}{u_{LL} + u_C w_L} < 0 \tag{80}$$

$$y_K = An^{-\nu}(F_K + F_L L_K) = An^{-\nu}\left(F_K + \frac{u_L}{u_{LL} + u_C w_L}F_{LK}\right) > 0 \tag{81}$$

$$y_n = -\nu An^{-\nu-1}F(K,L) + An^{-\nu}F_L L_n \tag{82}$$

$$= An^{-\nu-1}\left[-\nu F(K,L) + \frac{u_L}{u_{LL} + u_C w_L}(1-\nu)F_L\right] \tag{83}$$

$$= An^{-\nu-1}\left[-(F_K K + F_L L) - \frac{u_L}{u_{LL} + u_C w_L}(F_{LL}L + F_{LK}K)\right] \tag{84}$$

$$= An^{-\nu-1}\frac{1}{u_{LL} + u_C w_L}\left[-(F_K K + F_L L)(u_{LL} + u_C w_L) - u_L(F_{LL}L + F_{LK}K)\right] \tag{85}$$

$$= An^{-\nu-1}\frac{1}{u_{LL} + u_C w_L}\left[-\nu F u_{LL} - (F_K K + F_L L)u_L\frac{F_{LL}}{F_L} - u_L(F_{LL}L + F_{LK}K)\right] \tag{86}$$

$$= An^{-\nu-1}\frac{u_L}{u_{LL} + u_C w_L}\left[-\nu F(K,L)\frac{u_{LL}}{u_L} + K\left(\frac{F_{LL}}{F_L}F_K - F_{LK}\right)\right] < 0 \tag{87}$$

Use that $\nu F = F_K K + F_L L$ and $(\nu - 1)F_L = F_{LL}L + F_{LK}K$ and $u_C w_L = u_C w\frac{F_{LL}}{F_L} = -u_L\frac{F_{LL}}{F_L}$ since $u_C w = -u_L$.

D.1.3 Rents

$$r = (1-\zeta)An^{1-\nu}F_K(K,L) = (1-\zeta)\frac{\partial Y}{\partial K} = (1-\zeta)n\frac{\partial y}{\partial K} \quad (88)$$

$$r_C = (1-\zeta)An^{1-\nu}F_{KL}L_C = (1-\zeta)ny_C\frac{F_{KL}}{F_L} < 0 \quad (89)$$

$$r_K = (1-\zeta)An^{1-\nu}[F_{KK} + F_{KL}L_K] < 0 \quad (90)$$

$$= (1-\zeta)An^{1-\nu}[F_{KK} + \frac{u_L}{u_{LL} + u_C w_L}\frac{F_{LK}^2}{F_L}] \quad (91)$$

$$= (1-\zeta)An^{1-\nu}\frac{1}{u_{LL} + u_C w_L}[F_{KK}(u_{LL} + u_C w_L) + u_L\frac{F_{LK}^2}{F_L}] \quad (92)$$

$$= (1-\zeta)An^{1-\nu}\frac{1}{u_{LL} + u_C w_L}[F_{KK}(u_{LL} + u_C w\frac{F_{LL}}{F_L}) + u_L\frac{F_{LK}^2}{F_L}] \quad (93)$$

$$= (1-\zeta)An^{1-\nu}\frac{1}{u_{LL} + u_C w_L}[F_{KK}(u_{LL} + u_C w\frac{F_{LL}}{F_L}) - u_C w\frac{F_{LK}^2}{F_L}] \quad (94)$$

$$= (1-\zeta)An^{1-\nu}\frac{u_L}{u_{LL} + u_C w_L}\left[F_{KK}\frac{u_{LL}}{u_L} - \frac{(F_{KK}F_{LL} - F_{KL}^2)}{F_L}\right] < 0 \quad (95)$$

$$r_n = (1-\zeta)A[(1-\nu)n^{-\nu}F_K + n^{1-\nu}F_{KL}L_n] > 0 \quad (96)$$

$$= (1-\zeta)(1-\nu)y_K \quad (97)$$

We use the assumption that the production function satisfies the second partial derivative test for concavity: $F_{KK}F_{LL} - F_{KL}^2 > 0$ to establish $r_K < 0$. We also use $w_L = w\frac{F_{LL}}{F_L}$ and $u_L = -u_C w$.

D.1.4 Profit

$$\pi = y(1-(1-\zeta)\nu) - (1-\zeta)\nu\phi \quad (98)$$

$$\pi_C = y_C(1-(1-\zeta)\nu) < 0 \quad (99)$$

$$\pi_K = y_K(1-(1-\zeta)\nu) > 0 \quad (100)$$

$$\pi_n = y_n(1-(1-\zeta)\nu) < 0 \quad (101)$$

D.2 Eigenvalues, Determinacy, Saddle Stability

The characteristic polynomial associated with the Jacobian matrix can be determined through its principal minors.[10] Principal minors are those that correspond to the leading diagonal of the Jacobian matrix. Where M_k denotes the sum of principal minors of dimension k, the characteristic polynomial can be expressed as (Jacobson 2012, p. 196)

$$c(\lambda) = \det(\mathbf{J} - \lambda \mathbf{I}) = \lambda^4 - M_1\lambda^3 + M_2\lambda^2 - M_3\lambda + M_4 \tag{102}$$

where $M_1 = tr(\mathbf{J})$ and $M_4 = \det \mathbf{J}$. In the main paper, for the parameterized case, this equation has four solutions: two are positive (unstable) and two are negative (stable). We denote these eigenvalues

$$\lambda_1 \leqslant \lambda_2 < 0 < \lambda_3 \leqslant \lambda_4$$

In this section we derive the general minors and thus coefficents on the quartic polynomial for the general case.

All elements are evaluated at steady state. Denote $\det(\cdot)$ as $|\cdot|$.

[10]See section E.4 for a detailed discussion of deriving these principal minors in the parameterized case.

D.2.1 M_2

$$
\begin{aligned}
M_2 = M_{(1,1),(2,2)} + M_{(1,1),(3,3)} + M_{(1,1),(4,4)} \\
+ M_{(2,2),(3,3)} + M_{(2,2),(4,4)} + M_{(3,3),(4,4)}
\end{aligned} \tag{103}
$$

$$
\begin{aligned}
M_2 &= \begin{vmatrix} Y_K & Y_n \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} \rho & -\frac{\pi_n}{\gamma} \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} \rho & -\frac{\pi_K}{\gamma} \\ 0 & Y_K \end{vmatrix} \\
&\quad + \begin{vmatrix} \frac{C^*}{\sigma} r_C & \frac{C^*}{\sigma} r_n \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} \frac{C^*}{\sigma} r_C & \frac{C^*}{\sigma} r_K \\ Y_C - 1 & Y_K \end{vmatrix} + \begin{vmatrix} \frac{C^*}{\sigma} r_C & 0 \\ -\frac{\pi_C}{\gamma} & \rho \end{vmatrix}
\end{aligned} \tag{104}
$$

$$
= \frac{\pi_n}{\gamma} + \rho Y_K + \frac{C^*}{\sigma}(r_C Y_K - r_K(Y_C - 1)) + \frac{C r_C \rho}{\sigma} \tag{105}
$$

$$
= \frac{\pi_n}{\gamma} + \rho Y_K + \frac{C^*}{\sigma}(r_C(Y_K + \rho) - r_K(Y_C - 1)) \tag{106}
$$

$$
= \frac{\pi_n}{\gamma} + \rho \left(Y_K + \frac{C^*}{\sigma} r_C \right) + \frac{C^*}{\sigma}(r_C Y_K - r_K(Y_C - 1)) \tag{107}
$$

D.2.2 M_4

Use Laplace expansion, with row 4 elements multiplied by each of their corresponding cofactors. The zeros in row 4 simplify calculations.

$$
M_4 = \det(\mathbf{J}) \tag{108}
$$

$$
= 0 \cdot (-1)^{4+1} C_{(4,1)} + 1 \cdot (-1)^{4+2} C_{(4,2)} + 0 \cdot (-1)^{4+3} C_{(4,3)} + 0 \cdot (-1)^{(4+4)} C_{(4,4)} \tag{109}
$$

$$
M_4 = 1 \cdot (-1)^{4+2} \begin{vmatrix} \frac{C^* r_C}{\sigma} & \frac{C^* r_K}{\sigma} & \frac{C^* r_n}{\sigma} \\ \frac{-\pi_C}{\gamma} & \frac{-\pi_K}{\gamma} & \frac{-\pi_n}{\gamma} \\ Y_C - 1 & Y_K & Y_n \end{vmatrix} \tag{110}
$$

$$
= \frac{C^*}{\sigma \gamma} \left[(Y_C - 1)(r_n \pi_K - r_K \pi_n) + Y_K(r_C \pi_n - r_n \pi_C) + Y_n(r_K \pi_C - r_C \pi_K) \right] \tag{111}
$$

We can substitute out π_x, Y_x in terms of y_x where $x \in \{C, K, n\}$, dropping * notation

$$= \frac{ny(1-(1-\zeta)\nu)}{\sigma\gamma} [(ny_C - 1)(r_n y_K - r_K y_n) \tag{112}$$

$$+ny_K(r_C y_n - r_n y_C) + (y + ny_n)(r_K y_C - r_C y_K)] \tag{113}$$

$$= \frac{ny(1-(1-\zeta)\nu)}{\sigma\gamma} [r_K(y_n + yy_C) - r_n y_K - r_C y_K y] \tag{114}$$

Substitution of r_n gives

$$\frac{ny(1-(1-\zeta)\nu)}{\sigma\gamma} \left\{ \overbrace{r_K(y_n + yy_C)}^{(+)} \overbrace{-y_K[(1-\zeta)(1-\nu)y_K + r_C y]}^{(+/-)} \right\} \tag{115}$$

Therefore positivity of the second term is a sufficient condition for a positive determinant. This is simpler to show in the the parameterized case, which is what we do in the main paper, but can also be shown for the nonparametric case.

D.2.3 M_3

$$M_3 = M_{(1,1)} + M_{(2,2)} + M_{(3,3)} + M_{(4,4)} \tag{116}$$

$$M_3 = \begin{vmatrix} \rho & -\frac{\pi_K}{\gamma} & -\frac{\pi_n}{\gamma} \\ 0 & Y_K & Y_n \\ 1 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \frac{Cr_C}{\sigma} & \frac{Cr_K}{\sigma} & \frac{Cr_n}{\sigma} \\ Y_C - 1 & Y_K & Y_n \\ 0 & 0 & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} \frac{C^* r_C}{\sigma} & 0 & \frac{C^* r_n}{\sigma} \\ -\frac{\pi_C^*}{\gamma} & \rho & -\frac{\pi_n}{\gamma} \\ 0 & 1 & 0 \end{vmatrix} + \begin{vmatrix} \frac{C^* r_C}{\sigma} & 0 & \frac{C^* r_K}{\sigma} \\ -\frac{\pi_C}{\gamma} & \rho & -\frac{\pi_K}{\gamma} \\ Y_C - 1 & 0 & Y_K \end{vmatrix} \tag{117}$$

$$= (-1)^{3+1} \left(\frac{-\pi_K}{\gamma} Y_n - \frac{-\pi_n}{\gamma} Y_K \right) + 0 + (-1)^{3+2} \left(C^* \frac{r_C}{\sigma} \frac{-\pi_n}{\gamma} - C^* \frac{r_n}{\sigma} \frac{-\pi_C}{\gamma} \right) \tag{118}$$

$$+ (-1)^{3+1} (Y_C - 1) \left(-C^* \frac{r_K}{\sigma} \rho \right) + (-1)^{3+3} Y_K \left(C^* \frac{r_C}{\sigma} \rho \right) \tag{119}$$

$$= -\frac{\pi_K}{\gamma} Y_n + \frac{\pi_n}{\gamma} Y_K + C^* \frac{r_C \pi_n}{\sigma\gamma}$$

$$- C^* \frac{r_n \pi_C}{\sigma\gamma} - (Y_C - 1) \frac{C^*}{\sigma} r_K \rho + Y_K C^* \frac{r_C}{\sigma} \rho > 0 \tag{120}$$

D.2.4 M_1

$$M_1 = tr(\mathbf{J}) = \frac{C^* r_C}{\sigma} + \rho + Y_K \tag{121}$$

$$= 2\rho + \zeta Y_K > 0 \tag{122}$$

By Descartes' rule of signs and positivity of the determinant and trace, we can show that the model is a saddle. That is has two positive (stable) eigenvalues and two negative (unstable) eigenvalues. From left to right the coefficients of the characteristic polynomial are $+ \quad - \quad \pm \quad + \quad +$ Hence counting left to right there are two sign changes, regardless of whether M_2 is positive or negative (it is ambiguous in general). Thus there are at most two positive eigenvalues. There could be 2, 1 or 0 positive eigenvalues and conversely 2, 3 or 4 negative eigenvalues. Since the determinant is positive and the product of the eigenvalues, this singles out the 2 negative eigenvalues or 4 negative eigenvalue cases, and also rules out any eigenvalues equalling zero. We can rule out the four negative eigenvalues case by noting the trace is positive.[11]

D.3 Eigenvectors

To calculate the four eigenvectors corresponding to the eigenvalues $\lambda_j,\ j \in \{1,2,3,4\}$ solve $(\mathbf{J}-\lambda_j\mathbf{I})\mathbf{V}_j = 0$ for $\mathbf{V}_j$. By definition, the eigenvalues are chosen such that $\det(\mathbf{J}-\lambda_j\mathbf{I}) = 0$, and a zero determinant implies matrix $\mathbf{J}-\lambda_j\mathbf{I}$ is completely linearly dependent (perfectly coupled). Then the eigenvectors are unique only up to a scalar multiple. Hence choose $v_{4,j} = 1$ as the normalization. The result is normalized eigenvectors. Then by multiplying out $(\mathbf{J}-\lambda_j\mathbf{I})[v_{1,j} \quad v_{2,j} \quad v_{3,j} \quad 1]^\intercal = 0$, it is immediate from row four that $v_{2,j} = \lambda_j$. With $v_{2,j} = \lambda_j, v_{4,j} = 1$, we get from row 1 and 3

$$v_{1,j} = \frac{1}{\frac{C}{\sigma}r_C - \lambda_j}\left[\frac{-C}{\sigma}r_n - \frac{C}{\sigma}r_K v_{3,j}\right] \tag{123}$$

$$v_{1,j} = \frac{1}{Y_C - 1}\left[-Y_n - (Y_K - \lambda_j)v_{3,j}\right] \tag{124}$$

Equating and solving

$$v_{3,j} = \frac{\frac{C}{\sigma}r_n(Y_C - 1) - Y_n(\frac{C}{\sigma}r_C - \lambda_j)}{(\frac{C}{\sigma}r_C - \lambda_j)(Y_K - \lambda_j) - \frac{C}{\sigma}r_K(Y_C - 1)} \tag{125}$$

Plug back in

$$v_{1,j} = \frac{\frac{C}{\sigma}(r_K Y_n - r_n(Y_K - \lambda_j))}{(\frac{C}{\sigma}r_C - \lambda_j)(Y_K - \lambda_j) - \frac{C}{\sigma}r_K(Y_C - 1)} \tag{126}$$

[11] For a deeper discussion of complex and distinct (not repeated) roots of quartic equations see Rees 1922. Caputo 2005, p. 487 is a useful characterization of the 2d case.

So the four normalized eigenvectors are $\mathbf{V}_j = [v_{1,j} \quad v_{2,j} \quad v_{3,j} \quad 1]^\intercal, \quad j \in \{1,2,3,4\}$

$$\begin{bmatrix} \frac{\frac{C}{\sigma}(r_K Y_n - r_n(Y_K - \lambda_j))}{(\frac{C}{\sigma} r_C - \lambda_j)(Y_K - \lambda_j) - \frac{C}{\sigma} r_K (Y_C - 1)} \\ \lambda_j \\ \frac{\frac{C}{\sigma} r_n (Y_C - 1) - Y_n(\frac{C}{\sigma} r_C - \lambda_j)}{(\frac{C}{\sigma} r_C - \lambda_j)(Y_K - \lambda_j) - \frac{C}{\sigma} r_K (Y_C - 1)} \\ 1 \end{bmatrix} \tag{127}$$

This result replicates Brito and Dixon 2013 where there is perfect competition.

D.4 Solving the Model

By the stable manifold theorem, we set the constants of integration on the explosive eigenvalues to zero, then rearrange to give two saddle path conditions. These conditions on C, e ensure the economy is always on the stable set, given $K(t), n(t)$ (this is the closed loop solution form).

$$\begin{bmatrix} C(t) - C^* \\ e(t) - e^* \end{bmatrix} = \frac{1}{v_{3,1} - v_{3,2}} \begin{bmatrix} v_{1,1} - v_{1,2} & v_{1,1}v_{3,2} - v_{1,2}v_{3,1} \\ \lambda_1 - \lambda_2 & \lambda_1 v_{3,2} - \lambda_2 v_{3,1} \end{bmatrix} \begin{bmatrix} K(t) - K^* \\ n(t) - n^* \end{bmatrix} \tag{128}$$

The open-loop solution in terms of initial values K_0, n_0, time t and the parameters of the model Ω is

$$\mathbf{X}(t) = \mathbf{X}^* + a\mathbf{V_1}e^{\lambda_1 t} + b\mathbf{V_2}e^{\lambda_2 t}$$

where $\mathbf{X} \in \mathbb{R}^4$ is the state vector and $\mathbf{V_j}, \ j \in \{1,2\}$ is the normalized eigenvector associated with stable roots $\lambda_1 < \lambda_2 < 0$. The constants are $a = \frac{\hat{K} - v_{3,2}\hat{n}}{v_{3,1} - v_{3,2}}$ and $b = \frac{v_{3,1}\hat{n} - \hat{K}}{v_{3,1} - v_{3,2}}$ and $\hat{K} = K_0 - K^*$ and $\hat{n} = n_0 - n^*$, so in long-hand

$$C(t) = C^* + \frac{(\hat{K} - v_{3,2}\hat{n})v_{1,1}e^{\lambda_1 t} + (v_{3,1}\hat{n} - \hat{K})v_{1,2}e^{\lambda_2 t}}{v_{3,1} - v_{3,2}} \tag{129}$$

$$e(t) = e^* + \frac{(\hat{K} - v_{3,2}\hat{n})\lambda_1 e^{\lambda_1 t} + (v_{3,1}\hat{n} - \hat{K})\lambda_2 e^{\lambda_2 t}}{v_{3,1} - v_{3,2}} \tag{130}$$

$$K(t) = K^* + \frac{(\hat{K} - v_{3,2}\hat{n})v_{3,1}e^{\lambda_1 t} + (v_{3,1}\hat{n} - \hat{K})v_{3,2}e^{\lambda_2 t}}{v_{3,1} - v_{3,2}} \tag{131}$$

$$n(t) = n^* + \frac{(\hat{K} - v_{3,2}\hat{n})e^{\lambda_1 t} + (v_{3,1}\hat{n} - \hat{K})e^{\lambda_2 t}}{v_{3,1} - v_{3,2}} \tag{132}$$

Our transition experiments specify the initial capital and number of firms to steady state that arises under a different technology (e.g. $K_0 = K^*(A = 1.0), n_0 = n^*(A = 1.0)$) and then we study the solutions with $A = 1.01$ given this initial start position. The result is the solution paths from the old technology $A = 1.0$ to the new technology $A = 1.01$.

D.4.1 Long Run $t \to \infty$

Since the eigenvalues are negative (stable) the exponentials tend to zero as time gets large $e^{-\infty} \to 0$ then

$$C(\infty) = C^*, \quad e(\infty) = e^*, \quad K(\infty) = K^*, \quad n(\infty) = n^*$$

D.4.2 Short Run $t = 0$

$t = 0$ exponents are unitary

$$C(0) = C^* + \frac{(v_{1,1} - v_{1,2})\hat{K} + (v_{1,2}v_{3,1} - v_{1,1}v_{3,2})\hat{n}}{v_{3,1} - v_{3,2}} \tag{133}$$

$$e(0) = e^* + \frac{(\lambda_1 - \lambda_2)\hat{K} + (\lambda_2 v_{3,1} - \lambda_1 v_{3,2})\hat{n}}{v_{3,1} - v_{3,2}} \tag{134}$$

$$K(0) = K(0) \tag{135}$$

$$n(0) = n(0) \tag{136}$$

which emphasizes the jump propoerties of C, e and the quasi-fixity of K, n.

D.5 Consumption Behaviour

To understand how consumptions jumps in the short run, rewrite the closed-loop saddlepath conditions (128)

$$\begin{bmatrix} C(t) \\ e(t) \end{bmatrix} = \begin{bmatrix} C^* \\ e^* \end{bmatrix} + \begin{bmatrix} \Lambda_{1,1} & \Lambda_{1,2} \\ \Lambda_{2,1} & \Lambda_{2,2} \end{bmatrix} \begin{bmatrix} K(t) - K^* \\ n(t) - n^* \end{bmatrix} \tag{137}$$

The effect of a change in technology in general is

$$\begin{aligned} C(t)_A = C^*_A + \Lambda_{1,1_A}(K(t) - K^*) + \Lambda_{1,1}(K(t)_A - K^*_A) \\ + \Lambda_{1,2_A}(n(t) - n^*) + \Lambda_{1,2}(n(t)_A - n^*_A) \end{aligned} \tag{138}$$

Beginning in a neighbourhood of steady state removes the change in eigenvector terms, and as states are inert on impact, we also remove the change in state terms. Hence

$$C(0)_A|^* = C^*_A - \Lambda_{1,1}K^*_A - \Lambda_{1,2}n^*_A \tag{139}$$

The initial jump in consumption determines short-run labor response and therefore determines productivity undershooting or overshooting through capacity response. The response depends on the long-run movement of C, K, n and the slopes of the 2d stable manifold $\Lambda_{1,1}, \Lambda_{1,2}$.

E Parameterized Example

The baseline RBC model assumes isoelastic separable subutilties and a Cobb-Douglas production function.

$$U(C, L) = \frac{C^{1-\sigma} - 1}{1 - \sigma} - \xi \frac{L^{1+\eta}}{1 + \eta} \tag{140}$$

Taking labor as given, the derivatives are

$$U_C = C^{-\sigma} > 0, \quad U_{CC} = -\sigma C^{-\sigma-1} < 0, \tag{141}$$

$$U_L = -\xi L^{\eta} < 0, \quad U_{LL} = -\xi\eta L^{\eta-1} < 0 \tag{142}$$

Isoelastic utility implies there is constant elasticity of utility with respect to each good. The coefficient of relative risk aversion is constant $\sigma(C) = -C\frac{U_{CC}}{U_C} = \sigma$, and similarly $-L\frac{U_{LL}}{U_L} = -\eta$. Cobb-Douglas production conforms to our assumptions on the production function derivatives, and it is homogeneous of degree $\nu = \alpha + \beta$. The model is five equations in five variables. There is a static intratemporal condition that defines labor $L(C, K, n)$, and substituting this in gives four differentional equations in four variables $\{C, e, K, n\}$:

$$L(C, K, n) = \left(\frac{(1-\zeta)AK^{\alpha}\beta n^{1-(\alpha+\beta)}}{\xi C^{\sigma}} \right)^{\frac{1}{1+\eta-\beta}} \tag{143}$$

$$\dot{C} = \frac{C}{\sigma}\left[(1-\zeta)A\alpha K^{\alpha-1}L^{\beta}n^{1-(\alpha+\beta)} - \rho\right] \tag{144}$$

$$\dot{e} = (1-\zeta)A\alpha K^{\alpha-1}L^{\beta}n^{1-(\alpha+\beta)}e - \frac{1}{\gamma}\left(AK^{\alpha}L^{\beta}n^{-(\alpha+\beta)}(1-(1-\zeta)\nu) - \phi\right) \tag{145}$$

$$\dot{K} = n\left[AK^{\alpha}L^{\beta}n^{-(\alpha+\beta)} - \phi\right] - \frac{\gamma}{2}e^2 - C \tag{146}$$

$$\dot{n} = e \tag{147}$$

E.1 Parameterized Steady State

It is simpler to use per firm variables k and l. The intratemporal condition is

$$C^* = \left(\frac{\beta(1-\zeta)Ak^{*\alpha}l^{*\beta-1-\eta}n^{*-\eta}}{\xi} \right)^{\frac{1}{\sigma}} \tag{148}$$

The dynamical system in steady state defines the following four nullclines

$$\dot{C} = 0 \qquad \alpha k^{*\alpha-1} l(k^*, n^*)^{\beta} = \frac{\rho}{A(1-\zeta)} \tag{149}$$

$$\dot{e} = 0 \qquad k^{*\alpha} l(k^*, n^*)^{\beta} = \frac{\phi}{A(1-(1-\zeta))\nu)} \tag{150}$$

$$\dot{K} = 0 \qquad C^* = \frac{\phi(1-\zeta)\nu}{1-(1-\zeta)\nu} n^* \tag{151}$$

$$\dot{n} = 0 \qquad e^* = 0 \tag{152}$$

Therefore substituting $C^*(n^*)$ into the intratemporal condition gives

$$l^*(k^*, n^*) = \left(\frac{(1-\zeta)^{1-\sigma} A k^{*\alpha} \beta n^{*-\sigma-\eta}}{\xi \left(\frac{\phi\nu}{1-(1-\zeta)\nu} \right)^{\sigma}} \right)^{\frac{1}{1+\eta-\beta}} \tag{153}$$

Solving (149) and (150) gives k^* and l^*

$$k^* = \frac{\phi\alpha(1-\zeta)}{(1-(1-\zeta)\nu)\rho} \tag{154}$$

$$l^* = \left[\frac{1}{A} \left(\frac{\rho}{\alpha(1-\zeta)} \right)^{\alpha} \left(\frac{\phi}{1-(1-\zeta)\nu} \right)^{1-\alpha} \right]^{\frac{1}{\beta}} \tag{155}$$

Capital per firm is decreasing in market power, whilst labor per firm is ambiguous (this can be shown in general). Rearranging the intratemporal condition (153) gives[12]

$$n^* = \left(\left(\frac{\alpha}{\rho\nu} \right)^{\sigma} \frac{(1-\zeta)A\beta}{\xi k^{*\sigma-\alpha} l^{*1+\eta-\beta}} \right)^{\frac{1}{\sigma+\eta}} \tag{156}$$

[12] Intermediate step from rearranging intratemporal condition is $n^* = \left(\frac{l^{*1+\eta-\beta} \xi \left(\frac{\phi\nu}{(1-(1-\zeta)\nu)} \right)^{\sigma}}{k^{*\alpha}(1-\zeta)^{1-\sigma} A\beta} \right)^{-\frac{1}{\sigma+\eta}}$ and by noting $\left(\frac{\phi\nu}{(1-(1-\zeta)\nu)} \right)^{\sigma} = \left(\frac{\nu\rho k^*}{\alpha(1-\zeta)} \right)^{\sigma}$ simplifies to above.

Since k^* and l^* are increasing in overheads ϕ, then number of firms is decreasing in overheads. Substituting k^* and l^* gives

$$n^* = \left[\frac{\beta}{\xi\nu^\sigma}\left(A\left(\frac{1-(1-\zeta)\nu}{\phi}\right)^{1-\alpha}\left(\frac{\alpha(1-\zeta)}{\rho}\right)^{\alpha}\right)^{\frac{1+\eta}{\beta}} \left(\frac{\phi(1-\zeta)}{(1-(1-\zeta)\nu)}\right)^{1-\sigma}\right]^{\frac{1}{\eta+\sigma}} \tag{157}$$

Which simplifies to n^* in the paper. Or in terms of y^* this is

$$n^* = \left[\frac{\beta}{\xi\nu^\sigma}\left\{\left[\left(\frac{\alpha}{\rho}\right)^{\alpha}\frac{A}{y^*+\phi}\right]^{1+\eta}[(y^*+\phi)(1-\zeta)]^{\nu+\alpha\eta-\beta\sigma}\right\}^{\frac{1}{\beta}}\right]^{\frac{1}{\eta+\sigma}} \tag{158}$$

The factor price for capital is immediate (and general), whereas wage is function specific

$$r^* = (1-\zeta)AF_k^* = \frac{\rho}{1-\zeta} \tag{159}$$

$$w^* = (1-\zeta)AF_l^* = \beta\left[A\left(\frac{1-(1-\zeta)\nu}{\phi}\right)^{1-\nu}\left(\frac{\alpha}{\rho}\right)^{\alpha}(1-\zeta)^{\nu}\right]^{\frac{1}{\beta}} \tag{160}$$

Wage is convex in $1-\zeta$ so there is a w^*-maximizing market power, and similarly an n^*-maximizing market power.

E.2 Parameterized Steady State Comparative Statics

Without specifying functional forms, we already know in general

$$C_A^* = y^* n_A^* \tag{161}$$

$$y_A^* = 0 \tag{162}$$

$$l_A^* < 0 \tag{163}$$

with functional forms we can add

$$k_A^* = 0 \tag{164}$$

$$n_A^* = \frac{1+\eta}{\beta(\eta+\sigma)}\frac{n^*}{A} > 0 \tag{165}$$

$$K_A^* = k^* n_A^* > 0 \tag{166}$$

$$L_A^* = \frac{1-\sigma}{\beta(\sigma+\eta)}\frac{L^*}{A} \gtreqless 0 \iff \sigma \lesseqgtr 1, \qquad \sigma \in (0,\infty) \tag{167}$$

since $n_A^* > 0$ this means $C_A^* > 0$ and $K_A^* > 0$. The long-run effect of a change in technology on labor depends on σ. The result follows from using $L^* = l^* n^*$ and factoring out A. When $\sigma = 1$ then L^* is not a function of A since income and substitution effects cancel out. This corresponds to logarithmic utility which we assume for our simulations. They verify that a permanent change in technology causes no deviation in long-run labor supply from its original level. $\sigma > 1$ implies L^* is decreasing in A, since the income effect dominates the substitution effect. $\sigma < 1$ implies L^* is increasing in A, since the substitution effect dominates the income effect effect.

E.3 Parameterized Jacobian

In general from $y = An^{-\nu}K^{\alpha}L^{\beta} - \phi$ and our labor derivatives we can derive output responses for any t:

$$y_C = \beta(y+\phi)\frac{L_C}{L} = \frac{-\beta\sigma(y+\phi)}{1+\eta-\beta}\frac{1}{C} \tag{168}$$

$$Y_C = ny_C = \frac{-\beta\sigma(y+\phi)}{1+\eta-\beta}\frac{n}{C} \tag{169}$$

$$y_K = \frac{\alpha(y+\phi)}{K} + \beta(y+\phi)\frac{L_K}{L} = \frac{y+\phi}{K}\frac{\alpha(1+\eta)}{1+\eta-\beta} \tag{170}$$

$$Y_K = ny_K = \frac{y+\phi}{k}\frac{\alpha(1+\eta)}{1+\eta-\beta} \tag{171}$$

$$y_n = -\nu\frac{y+\phi}{n} + \beta(y+\phi)\frac{L_n}{L} = -\left(\frac{y+\phi}{n}\right)\left(\frac{\alpha+\nu\eta}{1+\eta-\beta}\right) \tag{172}$$

$$Y_n = y + ny_n = (1-\nu)(y+\phi)\left(\frac{1+\eta}{1+\eta-\beta}\right) - \phi \tag{173}$$

These forms are simple to understand in steady state as we have neat expressions for k^* and y^* and $\frac{n^*}{C^*} = \frac{1}{y^*}$. Hence substituting out of the Jacobian (72) gives

$$\begin{bmatrix} -\frac{\rho\beta}{1+\eta-\beta} & 0 & \frac{-\rho^2\nu(1-\nu+\eta(1-\alpha))}{(1+\eta-\beta)\sigma\alpha} & \frac{\phi(1-\zeta)\nu\rho(1-\nu)(1+\eta)}{(1+\eta-\beta)(1-(1-\zeta)\nu)\sigma} \\ \frac{(1-(1-\zeta)\nu)\beta\sigma}{(1+\eta-\beta)\gamma n^*(1-\zeta)\nu} & \rho & \frac{-(1-(1-\zeta)\nu)\rho(1+\eta)}{(1+\eta-\beta)\gamma n^*(1-\zeta)} & \frac{\phi(\nu(1+\eta)-\beta)}{(1+\eta-\beta)\gamma n^*} \\ \frac{-\beta\sigma}{(1+\eta-\beta)(1-\zeta)\nu} - 1 & 0 & \frac{\rho(1+\eta)}{(1+\eta-\beta)(1-\zeta)} & \frac{\phi[-\zeta\nu(1+\eta-\beta)+\beta(1-\nu)]}{(1+\eta-\beta)(1-(1-\zeta)\nu)} \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{174}$$

for clarity we leave n^* in the second row since it is a large expression.

E.4 Parameterized Principal Minors

We denote $M_{(2,2),(4,4)}$ for an individual minor corresponding to the rows and columns that remain when elements corresponding to $(2,2)$ and $(4,4)$ are deleted. This example would be one of the the six possible order 2 minors

$$M_{(2,2),(4,4)} = \det \begin{bmatrix} \dot{C}_C & \square & \dot{C}_K & \square \\ \square & \square & \square & \square \\ \dot{K}_C & \square & \dot{K}_K & \square \\ \square & \square & \square & \square \end{bmatrix} \tag{175}$$

The sum of principal minors of order 1 is the trace

$$M_1 = \frac{1}{1-\zeta}\left[2\rho + \frac{\zeta(1+\eta)}{1+\eta-\beta}\right] > 0 \tag{176}$$

The sum of principal minors of order 2

$$M_2 = M_{(1,1),(2,2)} + M_{(1,1),(3,3)} + M_{(1,1),(4,4)} + M_{(2,2),(3,3)} + M_{(2,2),(4,4)} + M_{(3,3),(4,4)} \tag{177}$$

$$= 0 + \frac{-\phi(\nu(1+\eta)-\beta)}{\gamma n^*(1+\eta-\beta)} + \frac{\rho^2(1+\eta)}{(1-\zeta)(1+\eta-\beta)} + 0 + \frac{-\rho^2\left[\beta\sigma + \nu(1-\zeta)(1-\nu+(1-\alpha)\eta)\right]}{(1-\zeta)(1+\eta-\beta)\alpha\sigma} + \frac{-\rho^2\beta}{1+\eta-\beta} \tag{178}$$

$$= \frac{\rho^2}{1+\eta-\beta}\left[\frac{-\phi(\alpha+\eta\nu)}{\gamma n^*\rho^2} + \frac{1+\eta}{1-\zeta} - \frac{\beta\sigma(1+(1-\zeta)\alpha) + \nu(1-\zeta)(1-\nu+(1-\alpha)\eta)}{(1-\zeta)\alpha\sigma}\right] \lesseqgtr 0 \tag{179}$$

The sum of the principal minors of order 3

$$M_3 = M_{(1,1)} + M_{(2,2)} + M_{(3,3)} + M_{(4,4)} \tag{180}$$

$$= \frac{-\rho(1+\eta)\phi\nu}{\gamma n^*(1+\eta-\beta)} + 0 + \frac{\rho\beta\phi}{\gamma n^*(1+\eta-\beta)} \tag{181}$$

$$+ \frac{-\rho^3\left[\beta\sigma + \nu(1-\zeta)(1-\nu+(1-\alpha)\eta)\right]}{(1-\zeta)(1+\eta-\beta)\sigma\alpha} < 0 \tag{182}$$

$$= \rho\left[M_2 - \rho^2\left(\frac{1}{1-\zeta} + \frac{\zeta\beta}{1+\eta-\beta}\right)\right] \tag{183}$$

The sum of principal minors of order 4 is the determinant

$$M_4 = \frac{\rho^2\phi\beta\nu(\eta+\sigma)}{(1+\eta-\beta)\gamma\sigma\alpha n^*} > 0 \tag{184}$$

E.5 Special Case $\eta = 0$ and $\sigma = 1$

The case of indivisble labor $\eta = 0$ and logarithmic utility $\sigma = 1$ is commonly assumed in RBC literature and recent entry-business-cycle papers (Bilbiie, Ghironi, and Melitz 2012; Jaimovich and Floetotto 2008). Indivisible labor maximizes labor elasticities, and logarithmic utility implies income and substitution effects cancel out in the long run, so long-run labor supply is independent of technology. The resulting system is vastly simplified:

$$M_1 = \frac{1}{1-\zeta}\left[2\rho + \frac{\zeta}{1-\beta}\right] > 0 \tag{185}$$

$$M_2 = \frac{\rho^2}{1-\beta}\left[\frac{-\phi\alpha}{\gamma n^* \rho^2} + \frac{1}{1-\zeta} - \frac{\beta(1+(1-\zeta)\alpha) + \nu(1-\zeta)(1-\nu)}{(1-\zeta)\alpha}\right] \lesseqgtr 0 \tag{186}$$

$$M_3 = \rho\left[M_2 - \rho^2\left(\frac{1}{1-\zeta} + \frac{\zeta\beta}{1-\beta}\right)\right] < 0 \tag{187}$$

$$M_4 = \frac{\rho^2\phi\beta\nu}{(1-\beta)\gamma\alpha n^*} > 0 \tag{188}$$

$$n^* = \frac{\beta}{\xi\nu}\left[A\left(\frac{(1-\zeta)\alpha}{\rho}\right)^{\alpha}\left(\frac{1-(1-\zeta)\nu)}{\phi}\right)^{1-\alpha}\right]^{\frac{1}{\beta}} \tag{189}$$

E.6 Parameterized Consumption Behaviour

Use that $k_A^* = 0$ so $K_A^* = k^* n_A^*$ and $C^* = n^* y^*$ so $C_A^* = n_A^* y^*$, then short-run consumption behaviour is

$$C(0)_A|^* = [y^* - \Lambda_{1,1}k^* - \Lambda_{1,2}]n_A^* \tag{190}$$

where y^*, k^* and for n_A^* are given in section E.2. Whether a change in firms increases or decreases short-run consumption depends on the sign of the square brackets and therefore the slopes of the stable manifold $\Lambda_{1,1}, \Lambda_{1,2}$ relative to y^*. This will determine short-run labor responses, and therefore capacity responses which determine productivity undershooting or overshooting.

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